THE USE OF ELECTRONIC DATA PROCESSING TO PREDICT THE MOISTURE AND SALT FLOW AND DISTRIBUTION WITHIN DANISH SALAMI DURING THE DRYING PROCESS

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Setting up of Differential Equations for the flow of Moisture and Salt:

It is assumed that a sausage can be defined by the following figures:

D = diameter (m)

= specific gravity of the sausage mix (kilo/m.3)

Fa= fat content (kilo fat/kilo sausage mix)

Wa= water content (kilo water/kilo sausage mix)

Sa= salt content (kilo salt/kilo sausage mix).

The part of the sausage mix which is not fat, water or salt, is assumed to be protein:

Pr = protein content = 1 - Fa - Wa - Sa (kilo protein/kilo sausage mix).

Furthermore, it is assumed that the sausage is made of pure meat, pure fat and additives free from fat, and that the moisture and salt flow take place only in that part of the sausage mix containing no fat.

Moisture Flow:

It is assumed that the part of the sausage mix containing no fat can be considered a capillary-porous body and that the moisture flow can be described by the equation:

$$w = -K_w \cdot \frac{dX}{dr} \cdot A \quad (1)$$

where

w = flowing moisture (kilo/h)

K_w= moisture conductivity (kilo/m.2 . h) (see later)

X = moisture - protein ratio (kilo/kilo)

r = radius of the cross-section in question of the sausage (m)

A = area of the cross-section in question (m.2)

= That part of the cross-section A trough which the moisture flow takes place

(corresponds to the part of the sausage mix volume containing no fat, i.e. = 1 - Fa /no dimensions/)

The differential equation for moisture flow is set up by considering the moisture balance of a cylinder concentrically with the axis of the sausage and with radius r, thickness dr and length = 1m.

According to (1) the moisture flow "w" is positive when it takes place in an outward direction along the radius.

With radius r, A being 2 r:

$$w_r = - K_{wr} \cdot \frac{dX_r}{dr} \cdot 2\ r . \qquad (2)$$

Then, by definition, with radius r+ dr:

$$w_{r+dr} = w_r + \frac{dw_r}{dr} dr \qquad (3)$$

Admission of moisture to the cylinder per unit of time will be:

$$w_r - w_{r+dr} = - \frac{dw_r}{dr} dr \qquad (4)$$

Differentiating (2) gives:

$$w_r - w_{r+dr} = 2\ r \cdot \ K_{wr} \ \frac{d^2X_r}{dr^2} + \frac{1}{r} \frac{dX_r}{dr} + \frac{dX_r}{dr} \cdot \frac{dK_{wr}}{dr} \ dr \qquad (5)$$

Accumulation of moisture in the cylinder per unit of time "dt" will be:

$$2 \ \ r \cdot dr \cdot \ \ \cdot Pr \frac{dX_r}{dt} \qquad (6)$$

As (5) and (6) express the same, then:

$$\frac{dX_r}{dt} = \frac{\ }{\cdot Pr} \ K_{wr} \ \frac{d^2X_r}{dr^2} + \frac{1}{r} \frac{dX_r}{dr} + \frac{dX_r}{dr} \cdot \frac{dK_{wr}}{dr} \qquad (7)$$

which is the differential equation for moisture flow and distribution required.

Salt Diffusion:

It is assumed that the movement of salt takes place as diffusion in the liquid phase and that it can be described by the following expression:

$$s = -K_s - \ \ _s \cdot \frac{dY}{dr} \cdot A \cdot X \cdot Pr$$

where:

s = salt diffusion (kilo/h)

K_s = salt conductivity (m.2/h) (see later)

$_s$ = specific gravity of the salt solution (kilo/m^3)

Y = salt concentration of the solution (kilo salt/kilo solution)

r = radius of the cross-section in question (m)

A = area of the cross-svection in question (m.2)

X.Pr= that part of cross-section "A" which is filled with liquid and consequently diffusing salt.

The differential equation for salt diffusion is set up by considering a cylinder.

With radius "r", "A" being = 2 r

$$s_r = - K_s \cdot {}_s \cdot \frac{dY_r}{dr} \cdot 2 \quad r \cdot X_r \cdot Pr \qquad (9)$$

Then, by definition, with radius r + dr

$$s_{r+dr} = s_r + \frac{ds_r}{dr} \cdot dr \qquad (10)$$

The admission of salt to the cylinder per unit of time will be:

$$s_r - s_{r+dr} = - \frac{ds_r}{dr} \cdot dr \qquad (11)$$

Differentiating (9), K_s being considered constant, gives:

$$s_r - s_{r+dr} = + K_s \cdot {}_s \cdot Pr \cdot 2 \quad r \quad X_r \frac{d^2Y_r}{dr^2} + \frac{dX_r}{dr} + \frac{X_r}{r} \quad \frac{dY_r}{dr} \; dr \qquad (12)$$

The accumulation of salt in the cylinder per unit of time is then:

$$2 \quad r \times dr \times \quad \times Pr \times \frac{ds}{dr} r \, ;$$

where

S_r = salt content (kg salt/kg protein);

Since (12) and (13) express the same, then,

$$\frac{dS}{dt} r = K_s \frac{x}{} {}_s \quad X_r \times \frac{d^2Y}{dr^2} r + \frac{dX}{dr} r + \frac{X}{r} r \quad \frac{dY}{dr} r \; ; \qquad (14)$$

which is required equation for salt diffusion and distribution.

Sausage Characteristics

Moisture Conductivity, K_w.

As previously mentioned the moisture conductivity, K_w, is to a very high degree dependent on the water content.

The literature does not contain data concerning the moisture conductivity of meat products. For plant products e.g., potatoes and beechwood, measurements have been carried out by Görling, Krisher et al. Until more detailed information on meat products is available, we will use the above-mentioned data for plant products to produce the probability curve for meat. In the calculation, the fact that the diffusion in the sausage is not straight forward (because of the pieces of fat) is taken into account by the factor L_w.

Salt Conductivity, K_s.

As previously mentioned, it is assumed that the salt flow is taking place as diffusion in the liquid phase. The moisture conductivity will therefore be directly proportional to the diffusion factor "d", which expresses the diffusion rate of salt into water. At the same time the fact that the diffusion by no means follows a straight line must be taken into account and again a factor Ls is inserted so that we have,

$$K_s = \frac{d}{L_s} \qquad (15)$$

When $d = 4 \times 10^{-6}\ m^2/h$ and $L_s = 1.5$ then,

$$K_s = 2.67 \times 10^{-6}\ m^2/h;$$

At low moisture contents, i.e. X 1 the moisture flow will mainly take place as vapour diffusion as the moist areas are no longer adjoining. Consequently, we have to assume that the salt diffusion will stop at X = 1, so that K_s is fixed at 0 when X 1.

This assumption is probably necessary if the model is to describe phenomena like dried outer ring and salt crystallation on the surface of the sausages.

The curve for K_s is shown in Fig. 3.

Surface Water Activity, $_{sf}$

The surface water activity, $_{sf}$ is calculated as a function of the moisture content of the surface: $X_{sf} = X_{r = D/2}$ and the salt/water ratio $(S/X)_{sf} = (S/X)_{r = D/2}$.

Using known data for high X values, $_{sf}$ is calculated as follows:

$$_{sf} = 0.975 \, . \, 0.8 \, (S/X)_{sf} \quad \times \quad 1 - \exp(-2X_{sf}) \qquad (16)$$

With $(S/X)_{sf}$ as parameter, the $_{sf}$ function is shown in Fig. 4 with X_{sf} as variable.

Limiting Conditions

The solution of the differential equations (7) and (14) requires knowledge of the limiting conditions, i.e. a knowledge of the physical changes which occur on the surface of the sausage, $r = D/2$ and its centre, $r = 0$ during the drying process.

At the surface of the sausage it is assumed that the amount of moisture which - moving towards the surface of the sausage is equal to the amount of moisture which - depending on the outer conditions - diffuses away from the surface of the sausage through the layer of air directly adjacent to the sausage surface.

It is also assumed that there is no diffusion of salt at the surface of the sausage and no diffusion of salt or water at the centre of the sausage.

Preliminary Conditions

At the beginning of the drying process the moisture and salt contents are assumed to be the same in every part of the sausage.

Furthermore it is assumed that the temperature is the same throughout the sausage and that this temperature is the "wet bulb" temperature of the sausage, i.e. that temperature which corresponds to the air temperature and humidity and to the water activity of the sausage.

Transformation of the differential equations into difference equations

Having now set up the equations which apparently describe the physical changes during the drying process the actual solutions still remains.

This is done by transforming the differential equations into difference equations (according to Plank) when the solution is derived numerically. The differences r and t should, however, be chosen so that they meet the convergence requirements. Thereby, the errors which the approximate nature of the calculations introduces will be eliminated and a fully satisfactory solution of the equations will be obtained.

Solution of the Equations

For the numerical treatment of the equations the Institute has made a computer programme in the language "Basic".

The result of a calculation is shown in Fig. 5 where also all data concerning the composition of the sausage and the drying conditions are given. See p. 11.

Discussion

We have now a mathematical model at our disposal which describes the drying process for a single salami sausage, and a computer programme which provides the numerical solution of the equations.

It is, of course, a prerequisite for its practical use that the model describes the actual conditions with sufficient accuracy.

Therefore, the next step is that the model should be tested experimentally so that, for instance, the estimated and actual losses can be compared. Fig. 5 shows such a comparison.

Since data for the moisture conductivity of potatoes and beechwood have been used for the model we have a certain amount of justification for undertaking certain modifications to the curve for the moisture conductivity. We think that this can be done by division with a factor and that this will be sufficient to make the model a true reflection of the physical reality.

Should, however, this not be the case, the model must be improved in the following ways:

1. By setting up a more correct differential equation for the movement of the salt which apart from the movement taking place by diffusion, also takes into account that salt is transported towards the outside of the sausage as a flowing salt solution.

2. By using smaller radius intervals, r (and time intervals, t) whereby the state at the surface of the sausage, which is so important for the drying process, is more accurately described.

3. By considering the changes in volume during drying.

Finally, more extensive research work might be carried out concerning the determination of moisture and salt distribution in sausages during drying and the determination of moisture and salt conductivities (cf. Görling).

In so far as the reliability of the model is concerned it has already been proved that it is not capable of describing a dry outer ring, a shortcoming which might originate from the fact that the radius intervals used (r = 1.5mm) were too large.

It is, of course, of enormous importance to be able to express the reasons for dry outer rings and the model should, therefore, by further improved.

With regard to the description of a normal drying process the present model does, however, seem satisfactory.

<u>Literature</u>

Imre, L. (1965-1966): Trocknungstechnische Analyse der Haltbarmachung von Salamiwurst. Zeitschrift für Lebensmittel-Untersuchung und -Forschung, 128. Band, Heft 3 und Heft 4.

Görling, O. (1956) : Untersuchungen zur Aufklärung der Trocknungsverhaltens pflanzlicher Stoffe. V.D.I.-Forschungsheft 458, Ausgabe B, Band 22.

Krischer, O. (1963) : Die wissenschaftlichen Grundlagen der Trocknungstechnik.

Plank, R (1959) : Handbuch der Kältetechnik, Dritter Band.

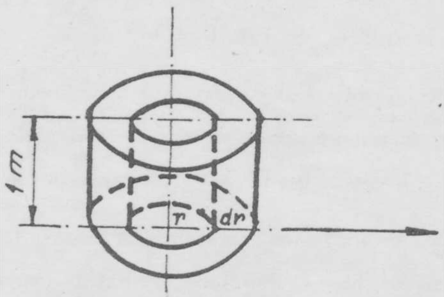

Fig. 1.

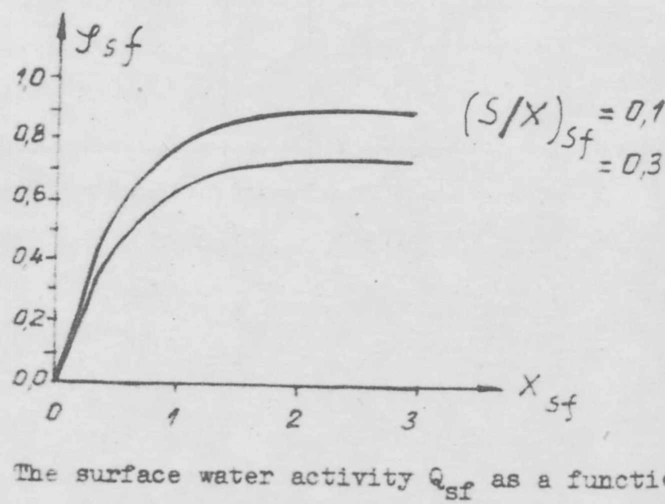

Fig. 4. The surface water activity Q_{sf} as a function of moisture content X_{sf} and salt/water ratio S/X_{sf}

When $L_w = 1.5$ the curve appears as shown in Fig.2.

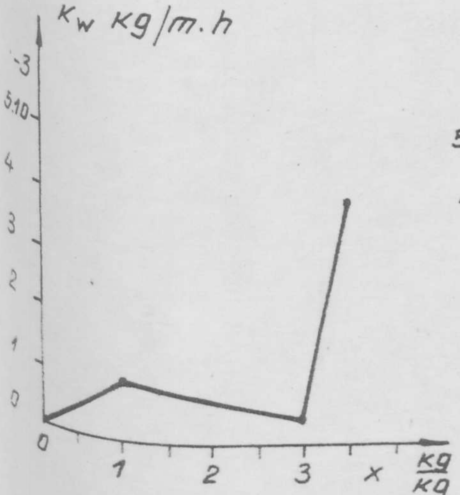

Fig. 2. Moisture conductivity K_w as a function of moi sture content X.

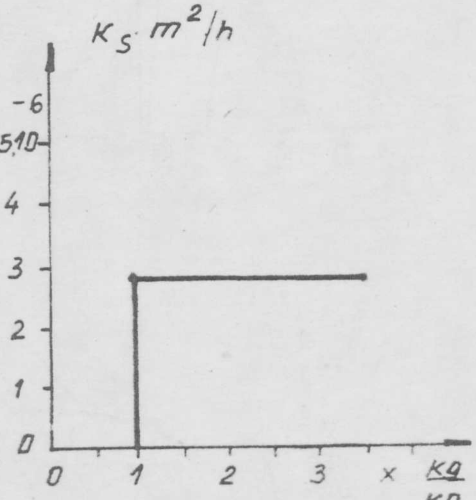

Fig.3 Salt condictivity K_s as a function of moisture content X

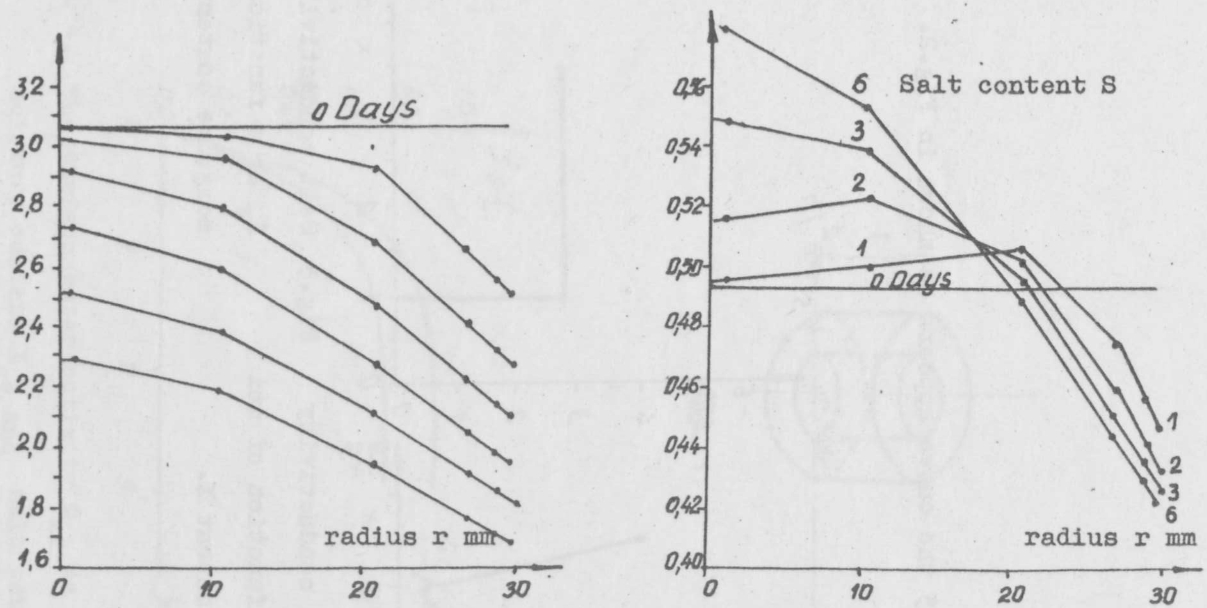

Fig. 5. Calculation of the drying process for salami sausages by means of the computer programme TO5TF. Air: 20 °C, 50% RH, α = 4 Kcal/m^{2}h°C.

Sausage: D= 60 mm, 39% fat, 41% water, 6.6% salt (NaCL).

Fig. 5: Calculation of the drying process for salami sausages by means of the computer programme T05TF. Air:20°C, 50% RH α= 4 Kcal/m^{2}h$^{\circ}$C. Sausage: D= 60mm, 39% fat, 41% water, 6.6% salt (NaCL).