Mathematical model for the manufacture of frankfurter type sausages

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Frankfurter type sausages are manufactured in Hungary on a large scale, therefore, it is essential that this be done in a profitable way. The cost of the raw materials has to be set at a minimum while still satisfying the requirements of chemical composition as specified in the Standard.
A mathematical model was developed permitting of the preparation of optimum formulae as a function of the chemical composition of the raw materials. The standard deviation arising in the raw materials and in the manufacture has been built in in the system of equation and thus these are not linear. The model may be used, apart from the development of formulae, in the modeling of theoretical problems and in profitability analyses.

Mathematisches Modell zur Herstellung von Brühwurstarten

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Die Herstellung von Brühwurstarten geschieht in Ungarn in reicher Sortenwahl, demzufolge ist es von grosser Bedeutung, die Herstellungsprozesse möglichst unter wirtschaftlichen Verhältnissen vorzunehmen. Einerseits müssen die Kosten der Rohstoffe aufs Minimum verringert werden, zur selben Zeit aber müssen noch die durch Normen bestimmten Bedingungen der chemischen Zusammensetzung erfüllt werden.
Das mathematische Modell wurde so formuliert, damit die Lösung des Modells eine Optimumrezeptur in Funktion der chemischen Zusammensetzung von Rohstoffen gibt.
Die in den Rohstoffen und während der Herstellung entstehenden Streuungswerte wurden in das Gleichungssystem eingebaut und daher sind sie nicht linear. Das Modell kann - neben der Rezepturgestaltung - bei der Modellierung von theoretischen Problemen und in den Wirtschaftlichkeitsanalysen verwendet werden.

13.5

Le modele mathématique de la production du saucisson type Francfort

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En Hongrie, on produit beaucoup de saucissons type. Francfort, c'est pous cela il est trés importont que la production soit économique. Les frais des matiéres premiéres doivent étre réduits au minimum, satisfaisont en menie temps les exigences des normes quont o. la composition chimique. Un modele mothématique a eté développé qui rend possible la diminirion des frais en fonction de la compositon chimique. Les écarts-type de la composition chimique de la muatiére preusiére ont été appliqués daus les équations qui devenarent ains non linéaires. Le modéle peut étre utilisé au but des considérations théoriques et pour augmenter l'économie de la production.

Математическая модель для производства бесструктурных колбас.

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Поскольку в Венгрии в большом количестве вырабатывают бесструктурные колбасы, поэтому большое значение имеет их экономическое производство. Расходы по сырью следует снизить до минимума, сохраняя при этом стандартный химический состав.
Разработана математическая модель для определения оптимальной рецептуры производства в зависимости от химического состава сырья.
Отклонения, вызываемые сырьём или процессом производства учитаны в уравнении, которое таким образом, является нелинейным. Кроме рецептуры для производства данная модель может быть использована для моделирования теоретических проблем и экономического анализа.

Mathematical model for the manufacture of frankfurter type sausages

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In Hungary the frankfurter type sausages are ranged with the "red" fresh sausages. Beside wieners the pariser is ranged in this group which is of a relatively large diameter and consumed cold. The polony /"krinolin"/ and knackwurst /"szafaládé"/ belong to this group, too, they are of medium diamter and are consumed, just as wieners, hot.

The common characteristic of these products is the emulsion used in them, thus their chemical composition and manufacturing technology is alike, too. The meat is pulped while adding polyphosphate, nitrite containing salt mixture, water and seasoning and finally some bacon. It is essential to achieve a homogeneous fine consistency, mostly by the use of cutter and fine cutter /e.g. Stephan/ /LŐRINCZ and LENCSEPETI, 1973/. Beside the sensory properties the standard chemical composition is the main quality requirement. To ensure this is necessary from the consumer aspect, on the one hand and in the economic interest of the manufacturer, on the other. Overstepping the minimum protein requirement, beyond the level justified by the scatter, causes a superfluous cost increase. The final composition is naturally mainly determined by the composiiton of the raw materials. Thus the knowledge of the initial parameters and the following up of changes during processing permits of the analysis of relations to the final composition. The model of manufacture of the product serves this purpose. If in the model developed the initial and final parameters are known /by analysis and by standard specification, respectively/ the dependent variables provide the optimum formula as a solution.

1. Basic relationships

1.1. Development of the system of equations

The quality of the products is, thus, laid down in the related standard. In developing the model the chemical composition and sensory properties as described in the standard have to be taken into account. The taking into account of the chemical characteristics seems to be the most simple. Since the quantity of ingredients /salt, polyphosphate, seasoning/ is constant, only the protein and water contents of the meat and bacon have to be accounted for. The standard specifications for fresh sausages are as follows /MSZ 5853-76/:

	in one sample	in the average of 4 samples
protein	9,5 %	10,5 %
water	70,5 %	69,5 %

For the sake of simplicity let us assume that the chemical composition of the sausage emulsion is identical with that of the ready made product and let us not consider the scatter either. If 1 kg emulsion is composed of α kg meat, β kg bacon and γ kg water, then, in the knowledge of percentage moisture and protein contentsof meat and bacon

/W_M, P_M, W_F, P_F/, the following equations are valid for the average of 4 samples:

$$\alpha P_M + \beta P_F \geq 10.5 \qquad \underline{a}$$

$$\alpha W_M + \beta W_F + 100\gamma \leq 69.5 \qquad \underline{b}$$

Since we calculate with 1 kg of emulsion and the emulsion contains about 2,5 % ingredient /salt + polyphosphate + seasoning/

$$\alpha + \beta + \gamma = 0.975 \qquad \underline{c}$$

Thus, the specifications of the standard in relation to chemical composoition were taken into account /ZUKÁL et al., 1964; KÁRPÁTI and ZUKÁL, 1967; ZUKÁL 1967; ZUKÁL and KÁRPÁTI, 1967/.

The most important characteristic of quality from the aspect of sensory value is the consistency of the fresh sausage, which depends besically on the water binding and emulsifying capacity of the meat utilized. This factor is a function of the quantitative and qualitative composition of protein. However, the relation between the protein quality and the water binding capacity of the meat cannot be described by a mathematical equation, inspite of the fact, that the problem has been discussed widely in the literature /HAMM, 1972/. This draws attention to the necessity of developing a method for the determination of the water binding capacity of meat showing a close correlation to the amount of water that may be added to the meat.

Thus, in the first approximation the addition of extraneous water is formulated as a function of the protein content of meat. On the basis of practical experience to a lean beef containing 21.4 % protein and 75 % water only 80 % extraneous water may be added; with meats of lower protein content /of higher fat content/ proportionately lower amounts. The water binding capacity of the protein content of bacon is neglected for the time being, since generally the conjunctive tissue contains a higher proportion of protein of lower water binding capacity:

$$\frac{80 + 75}{21.4} = 7,24$$

Thus:

$$\frac{\alpha W_M + 100\gamma}{\alpha P_M} \leq 7.24 \quad \text{and}$$

$$\alpha W_M + 100\gamma \leq 7.24\ \alpha P_M \qquad \underline{d}$$

The aim is to minimize the cost of the raw material required for the emulsion. If the emulsion is prepared from meat and bacon the target function is

$$C = \alpha C_M + \beta C_F \longrightarrow \text{min.} \qquad \underline{e}$$

where

C_M is the price of meat,

C_F is the price of bacon.

The price of water may be neglected.

Equations a and c are, in fact, conditional equations and the task belongs in the sphare of linear programming.

Since $\alpha + \beta + \gamma =$ constant, the value of γ in the equation may be substitited in equation b or d and thus the equation is of two variables and may be resolved graphically.

1.2. Example of calculation

Let us infer that the emulsion is made up of first class beef and bacon. The chamical composition of the raw materials is as follows: $W_M = 75$ %, $P_M = 21.4$ %, $W_F = 19$ %, $P_F = 6$ %.

$$\alpha\, 21.4 + \beta 6 \geq 10.5$$

$$\alpha\, 75.0 + \beta 19{,}0 + 100\gamma \leq 69.6 \qquad a'$$

$$\gamma = 0.975 - \alpha - \beta$$

$$\alpha\, 75.0 + \beta 19.0 + 97.5 - 100\alpha - 100\beta \leq 69.5$$

$$-25\alpha - 81\beta \leq -28 \qquad b'$$

$$\alpha 75 + 97.5 - 100\alpha - 100\beta \leq 7.24 \cdot \alpha \cdot 21.4$$

$$-179.9\alpha - 100\beta \leq -97.5 \qquad d'$$

And finally

$$\alpha + \beta \leq 0.975 \qquad c'$$

Taking a' b' c' and d' inequalities for equalities the curves are plotted /Fig. 1./

Fig. 1

It may be proven mathematically that it is suficient to look at the cost prices in the intersections and the composition of lowest cost is at the same time the minimum /KREKO, 1959; CSÁTH, 1972/. The beef was calculated at an average price of 71 Ft per kg and the bacon at 16 Ft per kg. All the possible combinations of compsoition, which satisfy the conditional equations fall in P_1, P_2, P_3 triangle.

The coordinates of the different points and the pertinent prime costs are /calculated by the target function/:

P_1	P_2	P_3
$\alpha = 0.88$	$\alpha = 0.43$	$\alpha = 0.31$
$\beta = 0.08$	$\beta = 0.22$	$\beta = 0.665$
$C = 63.75$ Ft/kg	$C = 34.05$ Ft/kg	$C = 32.65$ Ft/kg

1.3 Analysis of the solution of equations

The composition under P_3 is the cheapest, supposing the emulsion is prepared only of meat and becon without the addition of extreneous water. This is because the price of bacon is extremely low considering its protein content. Thus it follows that

the minimum prime cost does not go always with P_2 of maximum water content.

It is easy to prove that this occurs when

$$\frac{C_M}{C_F} > \frac{P_M}{P_F},$$

or the meat price- bacon price ratio is higher than the meat protein-bacon protein ratio /in this case 4.44 or 3.56/. If the protein content of bacon is lower, e.g. 4 %, in that case composition P_2 of maximum water content is the minimum.

The cost functions belonging to the constant prime cost were not illustrated in Fig.1, however, they may be easily plotted, since

$$\alpha = \frac{C}{C_m} - \frac{C_F}{C_M}\beta$$

Further, it is evident that if the protein content of bacon is about 10 % the minimum cost would be achieved by comminuting the bacon with the ingredients but without the addition of meat and extransous water.

However, this would be an absurditiy from the aspect of technology, thus the lower limit value of moisture content in fresh sausages must be also included in the conditional equations so as to ensure the character of the product. According to our present knowledge this is given as 62 %:

$$\alpha W_M + \beta W_F + 100\gamma \geq 62 \qquad \underline{f}$$

In the present example:

$$\alpha 75 + \beta 19 + 100\ /0.975 - \alpha - \beta/ \geq 62$$

$$-25\alpha - 81\beta \geq -35.5 \qquad \underline{f'}$$

Considering $\underline{f'}$ inequality as equality it was illustrated in Fig. 1. Besause of the high α value the point P_4 cannot represent minimum cost, only point P_5 /α = 0.40, β = 0.32, C = 33.52 Ft/kg/. Thus this is the composition representing minimum cost / γ = 0.975 - 0.40 - 0.32 = 0.255/.

Accordingly, 100 kg emulsion is composed as follows:

lean beef	40.0 kg
bacon	32.0 kg
water	25.5 kg
additives /salt, seasoning, polyphosphate/	2.5 kg
Total	100.0 kg

For the sake of simplification the calculations were related to 1 kg or 100 kg emulsion. The calculation is easier to survey if we say that 100 kg of emulsion give 98.04 kg final product, than if 102 kg goods. Theequation $\underline{d'}$, shown in Fig. 1, may be

neglected, since it is below points P_2 and P_5 marking maximum or minimum addition of water, respectively. This means that in this case only 75,6 % water may be added instead of 80 %, because of limiting condition b.

The expansion of the system of equations by accounting for the dispersion of the chemical composition is not included in this paper and is published elsewhere /Kürmendy and Erdős 1977; Kürmendy, Erdős, Zukál 1979/. However, it is to emphasize that the conditional equations do not remain linear in this case and the matematical solution is therefore more dificult and should be computerized.

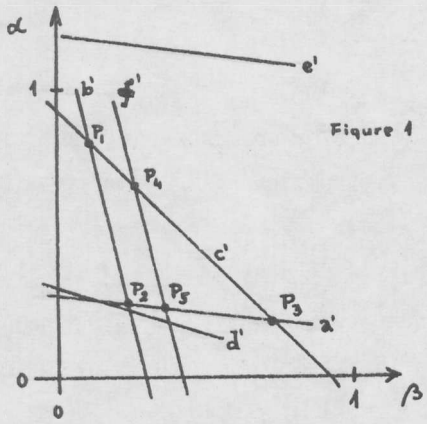

Figure 1

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