31 Predicting the effect of fat thickness and distribution on the heating times of joints of rolled meat

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Introduction

Mathematical models of the flow of heat and mass during cooking, freezing and thawing of meat are important because they provide an insight into the mechanismsinvolved. Many empirical equations have been developed to estimate processing times but their application is limited to the range of variables used in the experiments on which they are based. Analytical methods have more general application but are often restricted to cases where the material properties are assumed to be constant. Finite difference and finite element methods have been used to formulate and solve models involving foodstuffs with variable properties, with finite element techniquesbeing particularly suitable for irregularly shaped foods. Many models have been applied to freezing and thawing (James et al., 1977; Cleland and Earle, 1979a,b and Cleland et al., 1984) but fewer analyses are concerned with meat cooking; examples include Housova (1977), Sorenfors (1977), Dagerskog (1979a,b) and Burfoot and James (1983). These are often restricted to constant uniform thermal properties and/or one dimensional heat flow. The second limitation is obviously unjustified for most meat joints and reference to the literature (Morley, 1972) shows that the properties of fat, lean and bone differ considerably thereby invalidating the first assumption.

This paper presents a model, based on a finite cylinder, of the cooking of rolled beef joints in domestic conditions ("dry" air at 175°C) which allows for a cylinder of fat surrounding a lean cylinder or vice versa. Deficiencies of the model are considered and the effect of some of them is illustrated.

Mathematical model

The model uses Dusinberre's (1949) finite difference method, because it is conceptually simple, easily applied to regularly shaped objects and has previously proved useful (Bailey et al., 1974; James et al., 1977). The method involves dividing the object under study into a set of imaginary segments. For one dimensional heat flow through a single internal segment:

$$k_1 A_1 \left(\frac{\partial T}{\partial x}\right)_1 - k_2 A_2 \left(\frac{\partial T}{\partial x}\right)_2 = c\rho \frac{\Delta T}{\Delta t} \qquad [1]$$

Heat flow into - Heat flow out of = Rate of heat accumulation
the segment the segment

This equation assumes that the temperature gradients, $(\partial T/\partial x)$, are constant which is approximately true for small time intervals, Δt.

Bailey et al. (1974) and James et al. (1977) present the Dusinberre approximations for infinite slabs and cylinders with equal thermal conductivities at surfaces 1 and 2. Their equations are usual for cases with constant properties and heat transfer coefficient and with these restrictions the assumption of one dimensional heat flow can be eliminated using the Newman method (1931) which shows that

$$\left(\frac{T_a - T_{m,n}}{T_a - T_i}\right)_{\substack{\text{finite}\\ \text{cylinder}}} = \left(\frac{T_a - T_m}{T_a - T_i}\right)_{\substack{\text{infinite}\\ \text{slab}}} \cdot \left(\frac{T_a - T_n}{T_a - T_i}\right)_{\substack{\text{infinite}\\ \text{cylinder}}} \qquad [2]$$

where T_a, T_i = oven air temperature and initial uniform temperature of the meat and $T_{m,n}$, T_m, T_n = meat temperatures at position (m,n) in a finite cylinder, at longitudinal position m along an infinite slab and at radial position n through an infinite cylinder.

The methods described by Bailey et al. (1974) were formulated into a computer program for predicting the temperature distributions within infinite slabs and cylinders. The results of these predictions for a specific slab thickness and cylinder diameter were multiplied together, in the manner shown by equation 2, to calculate the temperature distribution within a homogeneous finite cylinder with the same length and diameter as those of the infinite slab and cylinder. The temperature distribution within a finite cylinder composed of a cylinder of lean surrounded by a cylinder of fat, or vice versa, was calculated using essentially the same method as that described above i.e. using equation 2. However, in this case, it is necessary to extend the work of Bailey et al. and to consider an infinite cylinder composed of two materials. At the interface of these materials, the Dusinberre approximations are

$$\frac{k_A 2\pi(r + \Delta r_A/2)l(T_{n-1} - T_n)}{\Delta r_A} - \frac{k_B 2\pi(r - \Delta r_B/2)l(T_n - T_{n+1})}{\Delta r_B}$$

$$\frac{\pi l[(r + \Delta r_A/2)^2 - r^2]\rho_A c_A + [r^2 - (r - \Delta r_B/2)^2]\rho_B c_B(T_n' - T_n)}{\Delta t} \qquad [3]$$

The temperature distribution at radial position, n, in an infinite two component cylinder can be calculated using this equation.

The model requires constant values of the properties of fat and lean; these were derived by evaluating the equations in Table 1 at 10°C intervals over the range 10 to 160°C and averaging these values i.e.

Material	Thermal conductivity $W\ m^{-2}K^{-1}$	Specific heat capacity $J\ kg^{-1}K^{-1}$	Density $kg\ m^{-3}$
Lean	0.4965	3947	982.4
Fat	0.1600	2795	898.2

Another factor needed in the model is the heat transfer coefficient. Natural convection coefficients acting over the curved surfaces of cylinders and over flat surfaces have been measured, but these have not been tested for application to rough surfaces, such as that of lean meat. However, the lack of alternative values for the present model justifies their initial use. The proposed equation is:

$$Nu = a(Gr.Pr)^m \qquad [4]$$

where Nu = Nusselt number = $h_c L/k$
Gr = Grashof number
and Pr = Prandtl number

Values of a and m for various ranges of (Gr.Pr) and curved and flat surfaces are reported by Perry and Chilton (1973).

During cooking, the transfer of heat by radiation is also considerable and, assuming the oven behaves as a black body, a radiation transfer coefficient, h_r, may be estimated using

$$h_r = \frac{\sigma\varepsilon(T_a^4 - T_s^4)}{(T_a - T_s)} \qquad [5]$$

where ε = emissivity of the meat surface; ε = 0.74 for lean beef; ε = 0.78 for beef fatty tissue (Miles, 1982).

The effective coefficient, h, is defined by

$$h = \frac{q}{A(T_a - T_s)} = h_c + h_r \qquad [6]$$

Equations 4 to 6 show that the coefficient, h, depends on the temperature difference between the air and the meat and on the dimensions of the meat. Both of these factors vary during cooking and invalidate an assumption of the model. A constant average h value of 18.2 W $m^{-2}K^{-1}$ was calculated using equations 4 to 6 for $(T_a - T_s)$ ranging from 15°C to 165°C at 10°C intervals. The properties of the air needed to calculate h_c were found using the correlations in Table 2. One further problem was the need to estimate the meat dimensions. Meat shrinks during cooking but the mechanismsgoverning shrinkage are insufficiently understood to allow calculations of the rate of shrinkage. A limited experimental investigation gave an indication of the likely changes. This involved cooking whole fat and lean cylinders, 160 mm long x 80 mm diameter to a centre temperature of 74°C in a natural convection oven at 175°C and the radial and longitudinal dimensional changes measured (Table 3). Since the simple model could not accept varying dimensions, the initial size was modified by multiplying the initial dimensions by the factors shown in Table 3. The results of these adjustments are presented later.

Results and discussion

Figure 1 illustrates typical predicted temperature profiles within a quarter portion of two joints. The larger temperature gradients within the fat, compared to those in the lean, are particularly noticeable and were also clearly apparent in the profiles for two component joints.

Table 4 presents cooking times predicted for various beef joints with or without fat at the surface or the centre. Although the model does not exactly simulate the geometry of rolled joints, one would expect to find similar trends regarding the effect of fat. The results indicate that beef joints with a surface fat layer cook slower than joints with an equivalent thickness of fat at the centre. Similarly with two joints of equal volume fractions of fat and lean, the joints with surface fat cook slower. However, one of the most interesting results is that the cooking times of these equally sized joints, with various amounts and distributions of fat, are all predicted to lie in the range 63 to 75 minutes.

The experimental cooking times shown in Table 4 were measured during the tests which provided the shrinkage data. Although for the whole fat cylinder, the difference between experimental and predicted cooking times is small the difference is considerable for the lean cylinder (-38%). This is particularly puzzling because the lean cylinder is exceptionally simple compared to most domestic joints; it has a regular shape, shrinks very little, and its published thermophysical properties appear almost insensitive to temperature. Furthermore, if the cooking times of this simple joint cannot be predicted accurately, it is clearly unwise, without further work, to accept predictions for the more complex two component joints. Obviously, we need to examine the reasons for the differences between experiment and prediction and use them to direct future research.

Certainly the model includes many sources of discrepancy between prediction and reality. Some are dependant on the oven, others on the material and the remainder caused by the interaction between the material and its environment. In this concluding section these sources are briefly discussed.

The model requires a single constant oven temperature, but in natural convection ovens temperature stratification occurs. In the shrinkage tests, four air temperatures, measured near to the meat, were all close to 175°C. However, at the top of the oven and near to the electrical heating elements, the temperature will be higher and such deviations from 175°C should reduce the cooking time below the predicted value. This is the opposite of the result found with the lean cylinder.

Dimensional changes during cooking have an important effect on cooking times (Burfoot and James, 1983) and must be accurately determined. However, the small dimensional changes of the lean cylinder were insufficient to account for the large difference between experimental and predicted cooking times. Analysis of the fat cylinder included other problems such as cracking and deformation into an oval cross section. These difficulties have been demonstrated by further experimental tests which showed good reproducibility of lean cooking times but large variations with fat.

Further to dimensional changes, the weight of meat changes during cooking. The effect of weight loss on cooking time is difficult to calculate because no heat and mass transfer models have been developed so far at this Institute. However, it is expected that evaporation losses, which require heat and may cause stationary boiling fronts, consequently increase the cooking times above the predictions.

In the model, the thermal properties of fat and lean and the heat transfer coefficient were assumed constant. Figure 2, which is considered in more detail later, clearly shows that the specific heat of fat, as estimated using the correlation in Table 1, varies during cooking. Similarly, equations 4 to 6 show that for the present conditions h also varies during cooking, although to a lesser extent (17.3 to 19.0 W $m^{-2}K^{-1}$). An enhanced model allowing for these variations predicted cooking times of 84 minutes (whole fat cylinder) and 67 minutes (whole lean cylinder). The large difference between the two predictions for fat cylinder cooking time reflects the importance of varying thermal properties. However, the enhanced model increased the difference between experimental and predicted fat cooking time. This may have been due to errors in the thermal properties and this was tested using differential scanning calorimetry with two 10 mg samples of the fat. Figure 2 shows the large divergence between the experimental and computed specific heat of fat and also the variation between samples. Furthermore, because none of the correlations in Table 1 are based on data above 90°C, although significant portions of joints are above this temperature during cooking, there may be further errors in the model. Similarly there may be errors in the heat transfer coefficients and these should be measured during cooking. Measuring h under natural convection conditions will be difficult and require a definition of the air temperature and the oven wall temperature which affects the radiation component of heat transfer.

This study has shown the problems in modelling meat during cooking. More relevant data, including heat transfer coefficients and high temperature thermophysical properties, are needed. The dimensional changes need to be measured and correlations developed for predicting them. Models should allow for temperature dependant properties and weight losses and these are being developed. Experimental investigations using PTFE cylinders will provide h values. Similarly, visual image analysis is being considered for investigating dimensional changes of meat. All of these studies are necessary for understanding the cooking process.

References

ASHRAE (1977) "1977 Fundamentals Handbook", 5.2.
Baghe-Khandan, M.S., Okos, M.R., Sweat, V.E. (1982) Trans. ASAE, 1118-1122.
Bailey, C., James, S.J., Kitchell, A.G., Hudson, W.R. (1974) J. Sci. Fd Agric., 25, 81-97.
Burfoot, D., James S.J. (1984) in Zeuthen, P. et al. "Thermal processing and quality of foods", Elsevier Applied Science, 467-472.
Cleland, A.C., Earle, R.L. (1979a) J. Food Sci., 44, 958-963, 970.
Cleland, A.C., Earle, R.L. (1979b) J. Food Sci., 44, 964-970.
Cleland, D.J., Cleland, A.C., Earle, R.L., Byrne, S.J. (1984) Int. J. Refr., 7, No. 1, 6-13.
Dagerskog, M. (1979a) Lebensm.-Wiss. u.-Technol., 12, 217-224.
Dagerskog, M. (1979b) Lebensm.-Wiss. u.-Technol., 12, 225-230.
Dusinberre, G.M. (1949) "Numerical analysis of heat flow", McGraw-Hill.
Housova, J. (1979) Proc. EFCE Minisymposium Math. Modelling in Food Processing, Orenas, Sweden, 249-267.
James, S.J., Creed, P.G., Roberts, T.A. (1979) J. Sci. Fd Agric., 28, 1109-1119.
Mason, E.A., Monchick, L. (1965) Chapter 4 in Wexler, A. (Ed.) "Humidity and moisture measurement and control in science and industry", Volume 3, 257-272.
Miles, C.A. (1982) "Heat transfer at the air/meat interface", Meat Research Institute Memorandum No. 53, Langford, Bristol, England.
Miles, C.A., van Beek, G., Veerkamp, C.H. (1983) in Jowitt, R. et al. "Physical properties of foods", Applied Science, 269-312.
Morley, M.J. (1972) "Thermal properties of meat: Tabulated data", Meat Research Institute Special Report No. 1, Langford, Bristol, England.
Newman, A.B. (1931) AIChE Trans, 27, 310-333.
Perry, J.H., Chilton, C.H. (1973) "Chemical Engineers' Handbook", McGraw-Hill, 10-10.
Riedel, L.(1955) Fette, Seifen, Anstrichmittel, 57, No. 10, 771-782.
Riedel, L.(1978) Chemie Mikrobiol. Technol. Lebensm., 5, No. 5, 129-133.
Sorenfors, P. (1977) Proc. EFCE Minisumposium Math. Modelling in Food Processing, Orenas, Sweden, 289-299.

Nomenclature

A = cross sectional area: c = specific heat capacity: Gr = Grashof number: h, h_c, h_r = effective, convection and radiation heat transfer coefficients: H = humidity: k = thermal conductivity: l = length: L = characteristic dimensional: Nu = Nusselt number: p = saturated vapour pressure: P = total pressure: Pr = Prandtl number: q = heat flow rate: r = radial position: t = time: T = temperature: T_a, T_i, T_s = oven air temperature, initial uniform meat temperature, and surface temperature: $T_{m,n}, T_m, T_n$ = meat temperatures at position (m,n) in a finite cylinder, at position m along an infinite slab and at radial position n through an infinite cylinder: $T_n' - T_n$ = temperature change at position, n, over time Δt: x = distance: x_F, x = mass fraction of fat and water: Δr = radial space increment: Δt = time increment: ΔT = temperature increment: ε = emissivity: ρ = density: σ = Stefan-Boltzman constant. [Subscripts: 1,2 = at planes 1 and 2: A,B = for materials A and B]

Figure 1 Typical predicted temperature profiles (in°C) within cylinders of fat and lean after 68 minutes of cooking in a natural convection oven at 175°C

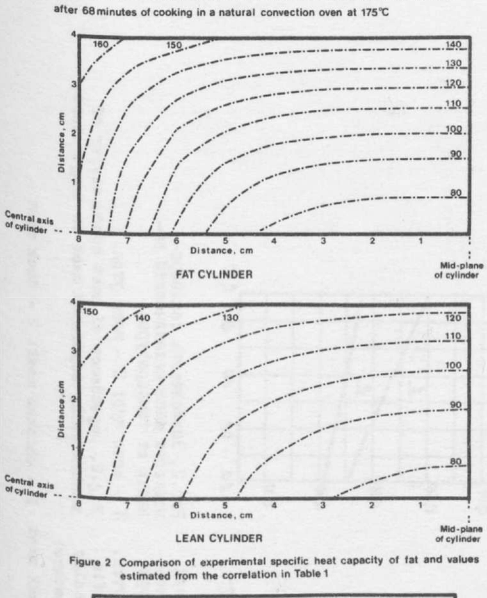

Figure 2 Comparison of experimental specific heat capacity of fat and values estimated from the correlation in Table 1

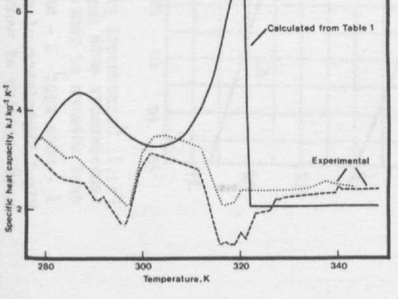

Table 1 Properties of beef lean and fat

Equations extracted from Baghe–Khandan et al. (1982)*, Riedel (1956)o, Riedel (1978)ø, Miles et al. (1983)□ or derived by regression analaysis of data from Baghe–Khandan et al. (1982)Δ, or derived by linear interpolation over the range 48 to 50°C∇

Lean

* $k = 0{\cdot}3037 - 0{\cdot}454x_F - 0{\cdot}219x + 0{\cdot}306T^{0{\cdot}039}$ W m^{-1} K^{-1}

ø $c = 1692 + 2575x + 4{\cdot}187(1-x)T - 335(1 - 0{\cdot}18xT)\exp(-43x^{2{\cdot}3})$ J kg^{-1} K^{-1}

Δ $\rho = 1027 - 0{\cdot}5257T$ kg m^{-3}

Fat

* $k = 0{\cdot}179 - 0{\cdot}000223T$ W m^{-1} K^{-1}

□ $$c = 1884 + 7{\cdot}008T + \frac{2280}{1 + \frac{0{\cdot}02(T-13{\cdot}8)^2}{2{\cdot}24}} + \frac{4280}{1 + \frac{0{\cdot}1(T-47{\cdot}85)^2}{4{\cdot}28}} \quad \text{J kg}^{-1}\text{K}^{-1} \quad \text{for } T<48°C$$

∇ $c = 6694 - 2322(T-48)$ J kg^{-1} K^{-1} for $48<T<50$°C

o $c = 2050$ J kg^{-1} K^{-1} for $T>50$°C

Δ $\rho = 934{\cdot}6 - 0{\cdot}4286T$ kg m^{-3}

Table 2 Properties of Moist Air

Equations extracted from ASHRAE Fundamentals (1977)* or derived from the results of Mason and Monchick (1973)o

* $$p = 165672{\cdot}4 \exp(0{\cdot}01/T')(374{\cdot}136 - T)\sum_{i=1}^{8} F_i(0{\cdot}65 - 0{\cdot}01T)^{i-1} \quad \text{mm Hg}$$

where

F_1 =	$-741{\cdot}9242$	F_5 =	$0{\cdot}1094098$
F_2 =	$-29{\cdot}72100$	F_6 =	$0{\cdot}439993$
F_3 =	$-11{\cdot}55286$	F_7 =	$0{\cdot}2520658$
F_4 =	$-0{\cdot}8685635$	F_8 =	$0{\cdot}05218684$
T =	temperature, K	T′ =	temperature, °C

o $\rho = 0{\cdot}3775(P - 0{\cdot}3780\,Hp/100)/T'$ kg m^{-3}

o $c = (1004{\cdot}832 + 1884{\cdot}06r)/(1+r)$ J kg^{-1} K^{-1}

where $$r = \frac{0{\cdot}62198\,Hp/100}{\left[P - \frac{Hp}{100}\right]}$$

o $$\mu = \left[1713{\cdot}27 + (4{\cdot}3T) - \left[\frac{184{\cdot}4\,Hp}{100P} + \frac{652{\cdot}5\,H^2p^2}{10^4P^2}\right] - \frac{0{\cdot}6567T\,Hp}{100P}\right].10^{-7} \quad \text{kg m}^{-1}\text{ s}^{-1}$$

o $$k = \left[509{\cdot}55 + (0{\cdot}45T) + \frac{206{\cdot}74\,Hp}{100P} - \frac{40{\cdot}83\,H^2p^2}{10^4P^2} + \frac{6{\cdot}04T\,Hp}{100P} - \frac{1{\cdot}84T\,H^2p^2}{10^4P^2}\right](4{\cdot}1868 \times 10^{-5}) \quad \text{W m}^{-1}\text{ K}^{-1}$$

Table 3 Dimensional changes of single cylinders of fat and lean.

Material	Initial length, I/mm	Final length, F/mm	% change of length (I-F)/F	Average length during cooking (I+F)/2,mm	Average length / Initial length
Lean	160·25	137·75	−14·0	149·00	0·9298
Fat	162·25	153·25	− 5·5	157·75	0·9723
Average for fat and lean					0·9510

Material	Initial diameter, I/mm	Final diameter, F/mm	%change of diameter (I-F)/F	Average diameter (I+F)/2,mm	Average diameter / Initial diameter
Lean	75·00	73·50	− 2·0	74·25	0·9900
Fat	80·00	46·00	−42·5	63.00	0·7875

The initial length of all the cylinders in the model were multiplied by the factor 0·9510. This simplified the model by neglecting longitudinal separation of fat and lean. The multiplying factors for shrinkage of diameter are as shown.

Table 4 Comparison of predicted and experimental cooking times of beef cylinders.

Nominal thickness of fat, mm	Fat thickness adjusted for shrinkage, mm	Fat location	Nominal thickness of lean, mm	Lean thickness adjusted for shrinkage, mm	Overall radius of cylinder, mm	Predicted cooking time, min	Experimental cooking time, min
0	0	—	40	39·6	39·6	67	107
7	5·51	Outside	33	32·67	38·18	70	—
17·5	13·78	Outside	22·5	22·28	36·06	75	—
33	25·99	Outside	7	6·93	32·92	71	—
40	31·50	—	0	0	31·50	68	63
33	6·93	Inside	7	25·99	32·92	68	—
22·5	17·33	Inside	17·5	17·72	35·04	67	—
7	32·67	Inside	33	5·51	38·18	63	—

Predictions were based on the following data:

Nominal length of cylinder	= 16cm
Length of cylinder, adjusted for shrinkage	= 15·2 cm
Initial uniform temperature of the cylinder	= 10°C
Temperature of the surrounding air	= 175°C
Heat transfer coefficient (based on air at 175°C, 760 mmHg and a relative humidity of 0·1%)	= 18·2 Wm^{-2} K^{-1}
Mass fraction of water in the lean	= 0·6891 } Based on the results for whole round, as found by Baghe–Khandan et al. (1982)
Mass fraction of fat in the lean	= 0·0379 }