PERFORMANCE OF A SIMPLE MATHEMATICAL MODEL FOR PREDICTING THERMAL CHANGES IN MEAT

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SUMMARY

A mathematical model of heat transfer in meat is described which can be programmed on a small personal computer. The model can be used in a wide range of chilling and freezing applications, including those where boundary conditions change with time. Predictions from the model are compared with published data and data measured by the Meat Research Laboratory in commercial beef chillers, air-blast carton freezers and plate freezers.

Temperatures calculated from the model agree well with measured data for beef sides of 48 kg to 222 kg, and for carton freezing times in the range 2 h to 38 h.

INTRODUCTION

A method for predicting cooling and freezing rates of beef sides and cartoned meat which utilises small personal computers is described. The beef side or meat carton is modelled as a one-dimensional meat slab convectively cooled at both surfaces. For beef sides a correlation is developed between the mass of the side and the thickness of the model slab which gives the same temperature history. Freezing is accounted for by varying the specific heat and thermal conductivity with temperature as suggested by Bonacina and Comini (1971). Temperatures in the slab are evaluated by the finite difference technique of Lees (1966), which uses three time-steps in each difference equation. This technique improves accuracy when the meat thermal properties vary with temperature. The finite difference equations for freezing calculations are solved by a computer program written in compiled Microsoft Basic®. The program runs on small personal computers and requires less than a minute for a 24 hour chilling or freezing prediction. The program does not incorporate refinements such as automatic time-step adjustment or heat balance checking, which can be found in more complex programs designed to run on larger computers (Cleland et al. 1984).

The model can simulate thawing of cartoned meat and unconventional chilling or freezing cycles such as those which use varying air temperatures and velocities.

EXPERIMENTAL METHODS

Description of the model

The model chosen for the present calculation is a one-dimensional slab of meat convectively cooled at its surfaces, as illustrated in Fig.1. Because the slab is symmetrical about the centre line, only one half of the slab needs to be considered. This half-slab has been divided into four equal elements of thickness Δx, and five nodes. For most chilling or freezing problems, Δx will have a value <20mm. The nodes are located at the centre of each element and at the surface. The surface element has zero mass to give more accurate surface temperatures. The conduction path between nodes (1) and (2) is therefore half of that between the other nodes. A time increment, Δt, of 4 seconds is used. The values of the time increment and node spacing are the optimum values which could be used. Larger values led to inaccurate and oscillatory solutions.

The general form of the Lees' approximation to the one dimensional equation for heat conduction is:

$$C_{h,i}(T_{h+1,i}-T_{h-1,i}) = \frac{2}{3}\frac{\Delta t}{\Delta x^2}\{k^+[(T_{h+1,i+1}-T_{h+1,i})+(T_{h,i+1}-T_{h,i})+(T_{h-1,i+1}-T_{h-1,i})]-k^-[(T_{h+1,i}-T_{h+1,i-1})+(T_{h,i}-T_{h,i-1})+(T_{h-1,i}-T_{h-1,i-1})]\} \quad \text{........(1)}$$

where $k^+ = (k_{h,i+1} + k_{h,i})/2$

and $k^- = (k_{h,i} + k_{h,i-1})/2$

The subscripts refer to time-step and node position respectively so that Th,i is the temperature at the time step h and node position i. Ch,i is the specific heat/unit volume. Δt and Δx are the time and node increments respectively, k^- is the thermal conductivity between the current node point and the adjacent node point closest to the surface, and k^+ is the thermal conductivity between the current node point and the adjacent node closest to the centre of the slab.

At the centre of the slab the node spacing is $\Delta x/2$, and $k^+ = 0$, so that:

$$C_{h,5}(T_{h+1,5}-T_{h-1,5}) = \frac{2}{3}\frac{\Delta t}{\Delta x^2}\{k^-[(T_{h+1,4}-T_{h+1,5})+(T_{h,4}-T_{h,5})+(T_{h-1,4}-T_{h-1,5})]\} \quad \text{........(2)}$$

At the convective surfaces the grid spacing is again x/2 and $k^- = H.\Delta x/2$ where H is the surface heat transfer coefficient (W/m^2 °C). Furthermore, since the surface element is chosen to have zero mass, the heat flow into this element during any time increment must equal the heat flow out. The equation for the surface element becomes:

$$H(T_{h+1,1}-T_{h+1,0}) + \frac{2k^-}{\Delta x}(T_{h+1,1}-T_{h+1,2}) = 0 \quad \text{........(3)}$$

where $T_{h+1,0}$ is the temperature of the convective medium.

The two equations for the slab boundaries (equations 2 and 3) and the equation for the interior points (equation 1) make up a set of simultaneous linear equations which are solved by the usual Gauss elimination method.

Carcass chilling

The slab model is used to predict the temperature history of the deep butt sides of beef. The convective heat transfer coefficient at the meat surface was obtained from the equation of Jurges (McAdams 1954 p.249) at a mean

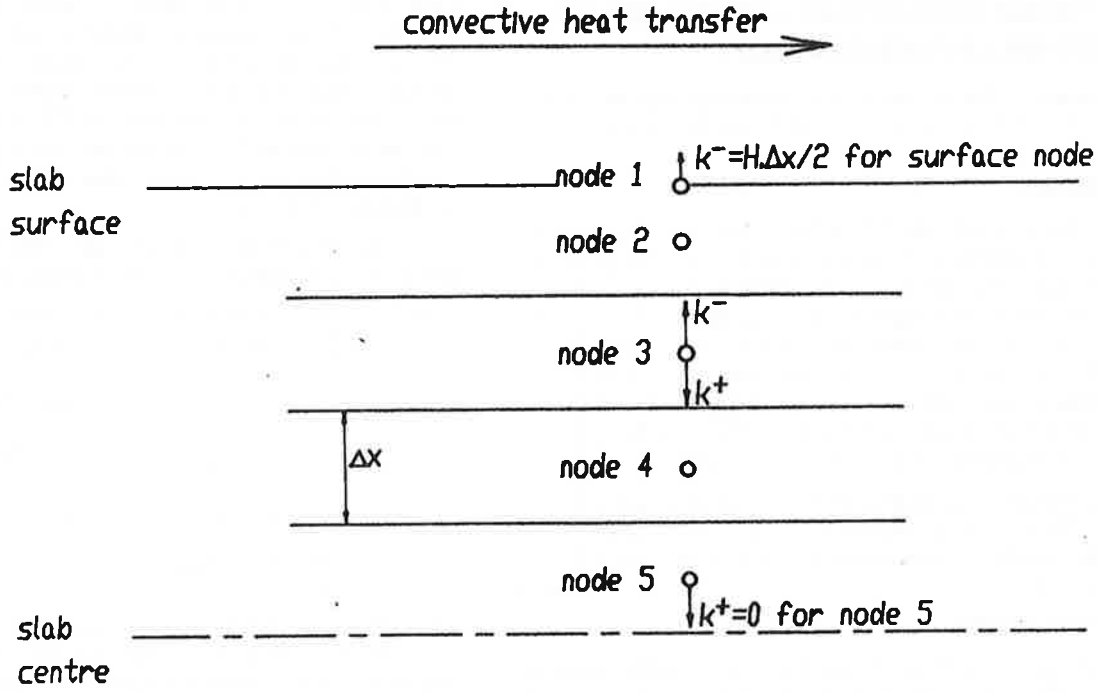

Fig.1 Finite difference representation of a meat slab.

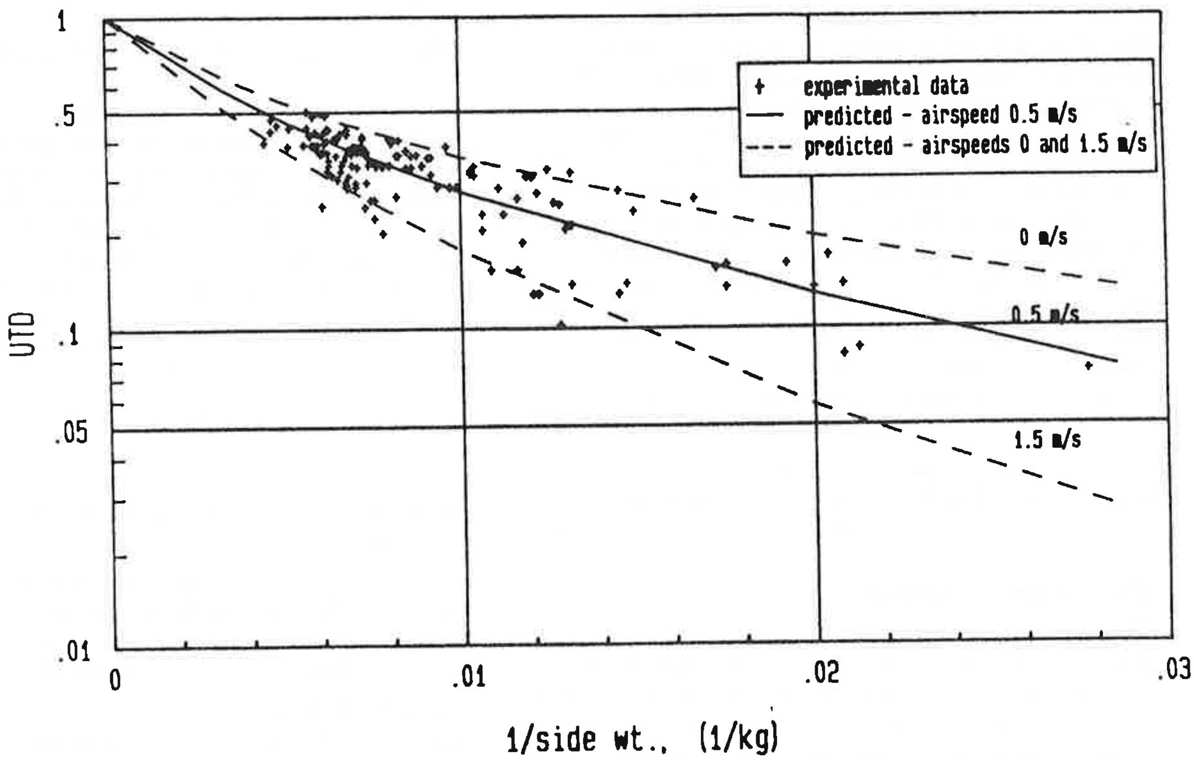

Fig. 2 UTD Values vs 1/(Side Weight) After 16 h.

TABLE 1: Calculated and measured freezing times for cartoned bulk meat

No. of cartons	Style	Board	Thickness mm	Air Vel. m/s	Initial temp. °C	Air temp. °C	Freezing time h Pred.	Freezing time h Meas.
4	[a]OSC	[c]Corr/B	152	8	9	-29.5	37.5	35-42.5
33	OSC	[c]Corr/B	152	8	15	-28.6	41.0	33-42.5
24	[b]B/L	[d]Solid	137	6	9	-29.0	25.0	22-29.5
5	B/L	[e]Solid	152	6.8	9	-28.0	29.0	26-29.5
5	B/L	eSolid	152	6	9	-28.5	30.0	28-31
8	B/L	[e]Solid	152	10	9	-29.0	27.0	25-31
1	B/L	[e]Solid	152	1.5	11	-31.0	34.0	33
1	B/L	Steel	152	1.5	11	-31.0	27.5	29

[a] overlap slotted container
[b] box and lid
[c] B-flute corrugated fibreboard, specification 293/293-117/1B
[d] solid fibreboard, specification 293/244-342/2
[e] solid fibreboard, specification 244/177-488/2

TABLE 2: Comparison of predicted freezing times with data measured on 'Karlsruhe test substance' by Cleland and Earle (1977)

Slab thickness mm	Surface HTC W/m² °C	Initial temp. °C	Air temp. °C	Freezing time to -10°C h Pred.	Freezing time to -10°C h Meas.
0.072	410	3.0	-22	2.1	1.92 ± 0.09
0.072	51.9	30.0	-40	2.6	2.62 ± 0.1
0.072	51.9	30.0	-20	5.3	4.80 ± 0.22
0.072	13.6	3.0	-26	8.9	8.42 ± 0.33

TABLE 3: Comparison of predicted and measured freezing times of cartons in plate freezers

No. of cartons	Style	Board	Thickness mm	Plate temp. °C	Initial temp. °C	Final temp. °C	Freezing time h Pred.	Freezing time h Meas.	Ref
9	B/L	Solid	144	-31	42	-7	17.0	16.6	1
1	B/L	Solid	165	-35	23	-8	17.2	19.5	2
20	B/L	Solid	144	-36	28	-8	13.7	11.6	3
1	OSC	Solid	150	-20	10	-12	27.0	26.0	4
1	OSC	Solid	150	-29	10	-12	18.3	21.0	4
1	OSC	Solid	150	-30	10	-12	18.0	18.0	4
1	B/L	Metal	150	-20	10	-12	14.0	11.0	4
1	B/L	Metal	150	-30	10	-12	9.0	10.0	4
1	B/L	Metal	150	-40	10	-12	8.0	6.0	4
1	OSC	Solid	150	-40	10	-12	14.0	11.0	4
1	B/L	Solid	162	-35	32	8	5.9	6.0	5
1	B/L	Corr/E	162	-35	32	8	5.9	6.3	5
1	B/L	Corr/B	162	-35	32	8	6.9	7.2	5
1	B/L	*Metal	162	-35	32	8	3.7	4.3	5
1	B/L	**Metal	162	-35	32	8	3.7	4.0	5
1	OSC	Corr/B	162	-35	32	8	7.5	8.5	5
1	B/L	Solid	162	-40	32	8	5.5	5.5	5
1	B/L	Corr/E	162	-40	32	8	5.5	5.8	5
1	B/L	Corr/B	162	-40	32	8	6.6	6.6	5
1	OSC	Corr/B	162	-40	32	8	7.2	7.8	5
1	B/L	*Metal	162	-40	32	8	3.6	4.0	5
1	B/L	**Metal	162	-40	32	8	3.6	3.7	5
1	B/L	*Metal	162	-25	32	8	4.6	5.5	5
1	B/L	**Metal	162	-25	32	8	4.6	5.1	5

Notes:
* Metal box with metal lid
** Metal box with no lid

References:
1. Anderson, Buhot, Larnach and Herbert (1984)
2. Anderson and Buhot (1982a)
3. Anderson and Buhot (1982b)
4. Downey (1980)
5. Visser (Personal Communication)

temperature of 20°C. Fat cover and evaporative heat loss are assumed to be zero. The relationship between the thickness of the slab and the mass of the side being modelled was obtained from comparison of calculated centre temperatures of slabs of different thicknesses after 20 h cooling, with measured deep butt temperatures of sides of different mass, also after 20 h cooling (Herbert et al. 1977). An average air velocity of 0.5 m/s was used.

The relationship determined from this comparison of 132 sides was:

$$\text{equivalent slab thickness (m)} = 0.012\sqrt{\text{side weight}} \quad \ldots\ldots\ldots(4)$$

The mass of the sides ranged from 36 kg to 224 kg with a mean of 124 kg. Air velocities ranged from 0.1 m/s to 1.5 m/s and air temperatures (averages of measured values at 8, 12, 16 and 20 h) ranged from 1.9°C to 9.6°C. Measurements were from 7 chillers.

Carton freezing

For cartoned meat the simulation is more geometrically accurate than for sides, particularly where cartons are pressed together side by side to effectively form a slab, as is the case in automatic air blast freezers and plate freezers. The heat transfer coefficient through the fibreboard, and from the fibreboard into the air has been calculated by adding the corresponding thermal resistances. The fibreboard resistance was obtained from Lovett et al. (1978), while the resistance between fibreboard and air was calculated from the convective heat transfer coefficients given by the equation of Jurges (McAdams 1954 p.249) at –20°C. For plate freezers the surface resistance was taken as that of the fibreboard or metal box, as a best estimate. Fibreboard resistance for overlap slotted container (OSC) cartons was assumed to be 1.25 times the single layer value to account for the partial double layer overlap on top and bottom surfaces. The average resistance to heat transfer of the air space between the top of the meat and the carton was found to be 0.029 m^2 °C/W for all types of carton in air blast freezers, and zero in plate freezers. The thermal properties of chilled and frozen meat were obtained from Morley (1972). The temperature dependence of the specific heat and thermal conductivity of the meat was accounted for by the method suggested by Bonacina et al. (1974).

RESULTS AND DISCUSSION

In Fig.2 the logarithm (to base 10) of the unaccomplished deep butt temperature difference (UTD = (deep butt temperature - air temperature)/(initial deep butt temperature - air temperature)) of 132 beef sides after 16 h chilling are plotted against the inverse of side weight. The variability of the UTD data at each mass is assumed to be caused by the variability of air velocity, and differences in fat thickness and side conformation between different sides. This form of plot was suggested by the Gurnie-Lurie diagrams (McAdams 1954) where log UTD vs $1/d^2$ gives almost linear plots for any convectively cooled body. The variable d is a characteristic dimension such as slab thickness. In Fig.2 d^2 has been replaced by side weight using the relationship given in equation 4.

Lines predicted by the model for air velocities of 0, 0.5 and 1.5 m/s are shown in Fig.3. An average of 95% of the data points were within 3°C of the 0.5 m/s line.

Predicted meat surface temperature was found to be always lower than the measured value. This reflects shortcomings in the model either in the surface heat transfer simulation (which as mentioned ignores evaporation and surface fat), or in the representation of the complex shape of the side by an infinite slab. Further work therefore is needed in this area.

Comparisons of predicted and experimentally determined freezing data (Lovett et al. 1978) are shown in Table 1, for cartons containing bulk meat in automatic air blast freezers.

Freezing times published by Cleland and Earle (1977) for 'Karlsruhe test substance' (a carboxymethyl cellulose product with thermal properties close to those of meat) are given in Table 2, together with values predicted by the model.

The results predicted by the model are slightly higher than the measured data of Cleland and Earle (1977). This may be due to errors in the values of the thermal properties, as Cleland and Earle do not give values for their trials. In any case, the errors in prediction are small compared with measured differences in freezing time in commercial freezing installations (see Table 1).

As expected, the one-dimensional model used for batch air blast freezers in this study predicts overlong freezing times for cartons where cold air can circulate over all faces of the cartons, rather than just two faces. Lovett and Herbert (1978) have concluded from measurements in commercial freezers that batch air blast freezers give freezing times to –12°C which are about 15% less than those measured in automatic air blast freezers. Freezing time predictions from the present model should therefore be reduced by about 15% for batch air blast freezers.

Table 3 gives a comparison of measured and predicted freezing times for fibreboard and metal cartons in plate freezers. The cartons contained bulk meat.

CONCLUSIONS

A simple one-dimensional mathematical model can be used to predict temperatures in beef sides and cartoned meat during chilling and freezing. Predictions were within 3°C of measured deep butt temperatures for 132 beef sides. The accuracy of prediction will be improved when the air velocity over the sides is known. Cartoned meat freezing time predictions were within 10% of measured values.

A computer program which solves the model equations can be run on many types of small personal computers and should be a useful research tool for evaluating different cooling and freezing regimes.

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