Simultaneous Heat and Mass Transfer During Beef Carcass Chilling

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SUMMARY: This paper describes the development of a heat and mass transfer model of a beef carcass chilling. The model includes thermal properties as functions of temperature and composition. The carcass was divided into 5 zones, round, sirloin, loin, rib and chuck; and carcass cross sectional structure within a zone was considered uniform. The heat and mass transfer in the vertical direction was neglected. The finite element formulation of the model is also described. Model was solved on an IRIS Workstation using FORTRAN language.

INTRODUCTION: The objectives of this study were: (i) to develop the heat and mass transport models for predicting the temperature and moisture profiles during beef carcass chilling, (ii) to validate the transport models using frankfurter data (Mittal, 1979), and, (iii) to generate temperature and moisture profiles for "round" section.

MODEL DEVELOPMENT: The model is based on the following assumptions: (i) whole carcass is heterogeneous and containes fatty tissue, bone and muscle only, (ii) thermal properties are functions of temperature and composition, (iii) moisture diffusivity is independent of direction of mass transfer, (iv) coupling of heat over mass and mass over heat on the molecular transfer is neglected, (v) no diffusive mass transfer in the frozen layer, (vi) the carcass was divided into 5 zones, round, sirloin, loin, rib and chuck; and carcass cross sectional structure was assumed to be uniform within a zone, and, (vii) the heat and mass transfer in the vertical direction was neglected.

The governing equations for heat and mass transfer are:

$$\frac{\partial}{\partial t}(\rho C T) = \frac{\partial}{\partial x}\left(K_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial T}{\partial y}\right)\ ; \qquad \frac{\partial M}{\partial t} = \frac{\partial}{\partial x}\left(D_m\frac{\partial M}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_m\frac{\partial M}{\partial y}\right)$$

The initial Conditions were $T = T_o$ and $M = M_o$ for $t \le 0$. Two boundary conditions are required, one for heat transfer and one for mass transfer and are explained below. The change in temperature at the surface is equal to the sum of the convective heat transfer at the surface and the evaporative heat loss due to latent heat of vaporization.

$$-A\left(K_x\frac{\partial T}{\partial x}n_x+K_y\frac{\partial T}{\partial y}n_y\right) = Ah(T_s-T_a)-V\rho_d L_v\frac{\partial M}{\partial t}$$

The change in moisture at the surface is equal to the convective mass transfer due to vapor pressure difference.

$$-D_m\rho_d\left(\frac{\partial M}{\partial x}n_x+\frac{\partial M}{\partial y}n_y\right) = k_m(P_s-P_a)$$

FINITE ELEMENT FORMULATION: The variational method was used to formulate the problem. The dependent variables M and T were approxiamted by interpolating functions of the form $M = \sum_{j=1}^{3} M_j(t)N_j(x,y)$ and $T = \sum_{j=1}^{3} T_j(t)N_j(x,y)$. Using the variational calculus method and setting the integral residual function equal to zero, the mass transfer equation becomes: $\int_V V\left(\frac{\partial M}{\partial t}-\nabla(D_m\nabla M)\right)dx.dy=0$. Using Green's theorem, the above equation was simplified to: $\sum_{j=1}^{n}\dot{M}_j[c_{ij}]+\sum_{j=1}^{n}M_j[k_{ij}]-[f_i]=0$;

where $[k_{ij}]=\int_V [B]^t[D][B]\,dx.dy$; $[c_{ij}]=\int_V \{N\}^t\lfloor N\rfloor\,dx.dy$ and $[f_i]=\frac{K_m}{\rho_d}(P_s-P_a)\int\{N\}^t ds$.

Using the same approach, the heat transfer equation was written as: $\sum_{j=1}^{n}\dot{T}_j[\tilde{c}_{ij}]+\sum_{j=1}^{n}T_j[\tilde{k}_{ij}]-[\tilde{f}_i]=0$;

where $[\tilde{k}_{ij}]=\int_V [B]^t[D^1][B]\,dx.dy+\int_L h\{N\}^t\lfloor N\rfloor\,ds$; $[\tilde{c}_{ij}]=\rho C\int_V \{N\}^t\lfloor N\rfloor\,dx.dy$ and $[\tilde{f}]=\left(h.T_a-\frac{V\rho}{A}L_v\frac{\Delta\bar{M}}{\Delta t}\right)\int\{N\}^t ds$;

$\bar{M}$ is the mass average moisture of the carcass. Also the matrices B, D and D^1 are defined as below:

$$[B_{ij}]=\begin{pmatrix}\frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x}\\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y}\end{pmatrix} \qquad [D]=\begin{pmatrix}D_m & 0\\ 0 & D_m\end{pmatrix} \qquad [D^1]=\begin{pmatrix}k_x & 0\\ 0 & k_y\end{pmatrix}$$

The global equations were obtained by assembling the element equations and writing them in a general form as

$[C(\phi)]\{\dot{\phi}\}+[K(\phi)]\{\phi\}+\{F\}=0$; *where*, $\{\phi\}=[M\ T]$; $[C(\phi)]$ *-Global mass matrix*; $[K(\phi)]$ *-Global stiffness matrix and* $\{F\}$ *-Global force vector*

SOLUTION SCHEME: The Crank-Nicolson central difference method was employed for marching through time because this method is second order accurate in time. Knowing the solution ϕ at time t, the solution at time (t+ dt) can be obtained by,

$$([C]+0.5*\Delta t[K])\phi_{t+\Delta t}=([C]-0.5*\Delta t[K])\phi_t+0.5*\Delta t(F_t+F_{t+\Delta t})$$

PHYSICAL PROPERTIES: The beef physical properties such as: thermal conductivity, specific heat capacity and density were calculated by using the models of Choi and Okos (1986). Surface heat transfer coefficient (h) was 35 W/(m^2.K) at an air velocity of 1 m/s and air temperature of 0°C, which was calculated by knowing Nu, Re, and Pr numbers (Kreith and Black, 1980). Surface mass transfer coefficient was determined by using Lewis relation and the ratio (h / (k_m L_v)) was 64.7 Pa/K for air velocities between 0.5 and 10 m/s (Daudin, and Swain, 1990). For this paper, the K_m was calculated as 14.35×10^{-12} (kg water.m/(kg DM.Pa.s)). The moisture diffusivity values were assumed based on the literature data for similar products. D_m for fat=3.07×10^{-11} m^2/s; for muscle=5.83×10^{-10} m^2/s; for bone = 5.48×10^{-12} m^2/s. The relationship for water activity at the surface in terms of moisture content and temperature was obtained by using the NLIN procedure of the Statistical Analysis Systems (SAS, 1988) and the data from Iglesias and Chirife (1982) as $Ln(a_w) - \frac{-842.827}{T} e^{(-14.578\,M)}$. Saturation vapor pressure was calculated using (Weiss, A., 1977): for 0<T<100 °C, P_s-$610.78\exp\left(\frac{17.269(T_{abs}-273.16)}{T_{abs}-35.86}\right)$, kPa and for -50<T<0 °C P_s-$610.78\exp\left(\frac{21.875(T_{abs}-273.16)}{T_{abs}-7.66}\right)$, kPa

Table I. Compositional Details for Round Section

Details	Water	Fat	Protein	Ash	Source
Fatty tissue	11.8	82.9	5.1	0.2	Koniecko (1979)
Muscle	73.0	4.8	21.2	1.0	Mc Keith et al., (1985)
Round Bone	26.0	30.0	20.8	23.2	Ockerman (1979)

Table II. Physical Properties for Frankfurter Simulation

k = 0.4306 W/(m.K)	C = 3.39 kJ/(kg.K)		L_v = 2326 kJ/kg
M_e = 0.4 d.b.			
h = 45.8333 W/(m^2.K)	ρ = 965 kg/m^3	ρ_d = 360 kg/m^3	M_i = 1.95 d.b.
D_m = 0.58×10^{-6} m^2/h	T_a = 69.0 °C	T_i = 22.5 °C	RH = 60%
Fat-Protein ratio =1.87			

FRANKFURTER SIMULATION: For moisture transfer in frankfurter simulation, the boundary condition was: M = M_e at [(X,y), (x,Y)] for t > 0. The simulations results were compared with the observed data points. The results are shown in figures 1 and 2 for moisture and temperature histories, respectively.

ROUND SIMULATION: The finite element layout for the round section is shown in Fig. 3. The maximum length and width of the section were assumed to be 22 cm and 15 cm. The air temperature was assumed to be 0°C and the air velocity was assumed as 1.0 m/s. The initial temperature of the carcass was assumed to be 30°C. The initial moisture content were obtained from the water content of the elements (from fat, bone, muscle data). Finer elements were laid around the boundary as the changes were rapid. Representative nodal points were selected to give more details on the temperature and moisture histories during chilling. The simulated moisture and temperature histories at the representative locations are shown in figures 4 and 5, respectively. The temperature at the surface was lowered to 0°C in 5 hours. The average centre temperature was 3.5°C in 10 hours. The surface moisture loss was found to varry with respect to locations. The difference was due to different fat layer thicknesses surrounding these locations. A loss of 83% and 72% of original m.c. was observed at locations I and VI (in Fig. 3). The average mass loss from the round was 0.05% after 10 hours of chilling.

CONCLUSIONS: From this study, the following conclusions were made: (1) The finite element model, validated by using the data for the frankfurter cooking, was in good agreement with the experimental data and predicted the temperature and moisture histories within ±2.5%. (2) The model predicted succesfully the temperature and moisture profiles during chilling a "round" section of a beef carcass. (3) Temperature at the surface was lowered to 0°C (location I and VI in Fig. 3.) in 5 hours; and the average centre temperature was 3.5°C after 10 hours. (4) Moisture loss from the carcass surface was 83% and 72% of original m.c. at locations I and VI in Fig. 3., respectively. The difference was due to different fat layer thicknesses surrounding these locations. (5) Average mass loss from the "round" was 0.05% after 10 hours.

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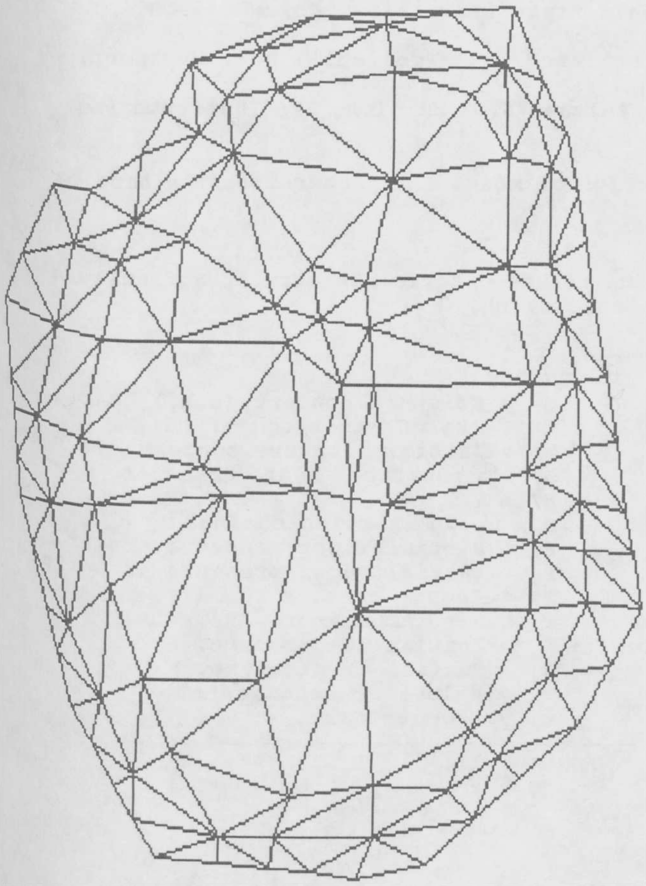

Fig.3. Finite element layout for round section

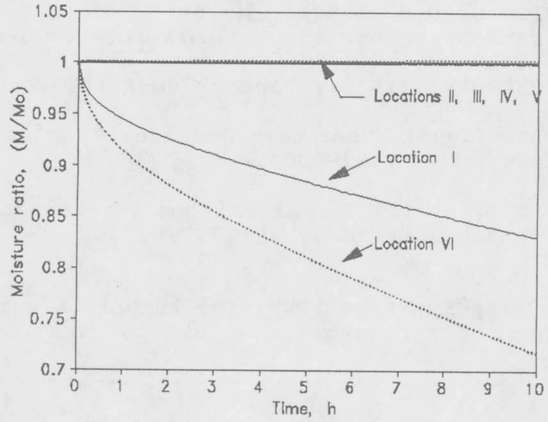

Fig. 4. Moisture profile during chilling

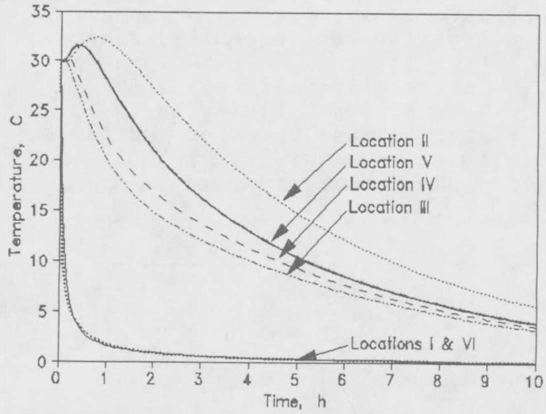

Fig. 5. Temperature profile during chilling

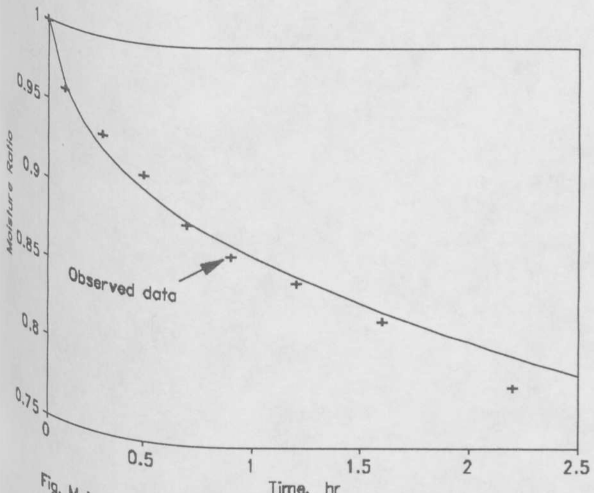

Fig. Moisture history of frankfurter for FP=1.87, RH=60%, Ta=69 C

Fig. 1. Moisture history of frankfurter.

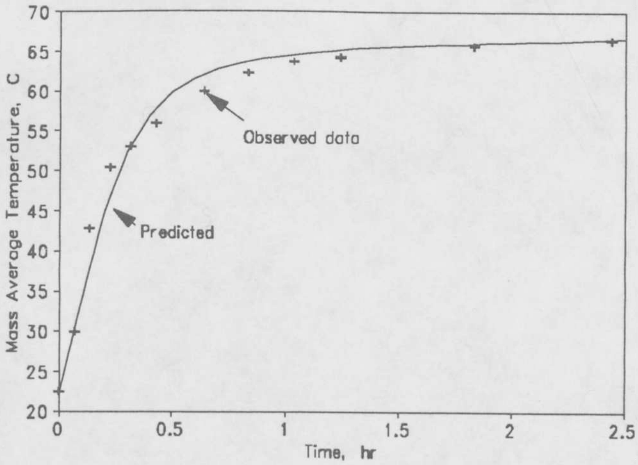

Fig. Temperature history of frankfurter for FP=1.87, RH=60%, Ta=69 C

Fig. 2. Temperature history of frankfurter

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LIST OF SYMBOLS

ρ = Bulk density, kg/m^3
ρ_d = Dry matter density, kg DM/m^3
A = Surface area $\perp$ to the direction of heat/mass flow, m^2
C = Specific heat capacity, J/(kg.K)
DM = Dry matter
D_m = Moisture diffusivity, m^2/s
k = Thermal conductivity, W/(m.K)
K_m = Surface mass transfer coefficient, kg of water/(Pa.s.m^2)
L_v = Latent heat of vaporization, J/(kg of Water)
M = Moisture content (d.b.), decimal, (kg of water/ kg of DM)
M_o = Initial moisture content, (d.b.)
h = Surface heat transfer coefficient, W/(m^2.K)
n_x, n_y = Direction cosines
P_a = Partial vapor pressure of air, Pa
P_s = Partial vapor pressure at surface, Pa
T = Temperature, K
T_a = Air temperature, K
T_o = Initial temperature, K
T_s = Surface temperature, K
V = Volume of carcass, m^3
x, y = Coordinates
