A SIMPLE WAY TO CALCULATE TEMPERATURE DISTRIBUTION IN AN INFINITE SLAB

DOORNBOS G.J.

Stork Bronswerk B.V., Amersfoort, The Netherlands

S-III.04

1. INTRODUCTION

There is a great need for easy to use mathematical formulae to solve technical problems. A difficulty is the complicated nature of the processes involved.
The cooling down of meat products is such a process.
The need for handy formulae arose in the working area of the author, calculating the temperature distribution of chicken carcasses during the cooling down process in chicken slaughter houses.

2. LITERATURE AND SCOPE

ID-DLO (former 't Spelderholt) in the Netherlands delivered a considerable amount of effort in describing the cooling process of chicken carcasses [1], [2], while the Meat Institute of TNO fulfilled this role in red meat industries [3].
Other sources are Levy [4] and the VDI Warmeatlas [5]. All authors calculate the mean temperature; Moerman calculates core temperatures, and Levy proposes a method to calculate the surface temperatures. Based on both proposals a formulae can be derived for the whole temperature distribution, core- and surface temperatures being extremes of this distribution.
To derive this result is the scope of this publication.

3. DERIVING THE TEMPERATURE DISTRIBUTION

3.1 Assumption

Following are the assumptions as accounted for:
-The cooled body can be seen as a slab.
-The body is isotrope, all physical proportion being the same throughout the slab.
-The initial temperature distribution is homogeneously throughout the slab.
- The derivation holds for times, sufficiently long to prevent calculation problems at the beginning of the cooling process. The cooling process can be characterized by two parameters, τ and j, and the temperature field is a second degree function of the distance.

The last assumption is now explained.

The temperature distribution can be written exactly [5]:

$$\frac{T-T_l}{T_b-T_l} = \sum_{i=1}^{\infty} \frac{2 \sin K_i}{K_i + \sin K_i \cos K_i} \cos(K_i \frac{y}{d}) \exp(-\frac{K_i^2 \, at}{d})$$

$$K_i = \frac{Bi}{\tan K_i}$$

This is an exact solution to the differential equation for an infinite slab.

Compared with this exact solution, it is investigated whether the calculated temperature distribution was sufficiently a square function.
Taking the Standard Process as an example (see for details fig. 1) a graph was constructed (fig. 2). At some cooling times the temperature profile is reported, together with the squared solution.
It is apparent, the differences are marginal, as can be seen in fig. 3. The solutions differ only some 0,8°C, rating less than about 4%.

It is concluded, that the assumption of a square temperature distribution is sufficient to describe the process.

3.2 Recapitulation of the first order cooling process

All sources state the process as follows:

$$T_g = T_l + (T_b - T_l)\, e^{-t/\tau} \qquad (1)$$

If both (T_b) (initial temperature) and (T_l) (ambient temperature) are known, the mean temperature (T_g) can be calculated as a function of time (t) by means of the parameter τ.

Expressed in dimensionless units, τ can be described as [4]:

$$\tau = \frac{d^2}{a.Bi}(1+0.423Bi) \qquad (2)$$

Where $Bi = \alpha d/\lambda$ en $a = \lambda/\rho c$. The distance d is measured as half the total thickness of the slab.
This is done because of the symmetry of the problem: the total thickness of the slab accounting for 2d.

The core temperature can be calculated by means of j [3]:

$$T_k = T_l + j(T_b - T_l)\,e^{-t/\tau} \qquad (1a)$$

The parameter j is constant during the precess (as well as τ) and has a value between 1 and 1.5. In definition:

$$j = \frac{1+Bi/2}{1+Bi/3} \qquad (3)$$

As can be seen, the j-value is a delay factor (1a) difined by (3).

3.3 Derivation of the j-value

When looking for a complete distribution of the temperature, we must generalize the j-value. Therefore, the next derivation is necessary.
Levy calculates the surface-, average- and coretemperature assuming a square temperature distribution in the slab, of the following form:

$$T - T_o = ay + by^2$$

T_o is the surface temperature and y is distance from the surface (y=0 at the surface, y=d at the core, see fig. 4). The constants a and b are determined from the conditions:

$$\frac{\delta T}{\delta y}\Big|_{y=d} = 0$$

$$\alpha\,(T_0 - T_i) = \lambda \frac{\delta T}{\delta y}\Big|_{y=o}$$

The result is:

$$T - T_o = (T_o - T_l)\,\frac{\alpha}{\lambda}\,(y - \frac{y^2}{2d}) = Bi\,(T_0 - T_l)\;Y \qquad (4)$$

Where Y = y/d - ½(y/d)². We try to express the surface temperatures in terms of the average temperature T_g. Levy states, that the place where this temperature T_g occurs is at:

$$\frac{y}{d} = 1 - \sqrt{1/3}$$

The place is given relative to the half slab width.
When this result is put in form (4), T_g can be solved:

$$T_g - T_o = Bi\,(T_o - T_l)\,[\,(1-\sqrt{1/3}) - \tfrac{1}{2}\,(1-\sqrt{1/3})^2\,]$$

This means:

$$T_o = \frac{3T_g + T_1 Bi}{3 + Bi}$$

Comparing with (4a) this result can be expressed as:

$$T - T_1 = (T_g - T_1)\frac{3+3BiY}{3+Bi} \qquad (5)$$

Again comparing with (1) and (1a), we recognize the second factor of the right hand side to be the generalized j-value to be

$$j_{gen} = \frac{3+BiY}{3+Bi} = \frac{3+3\left|\frac{y}{d} - ½\left(\frac{y}{d}\right)^2\right| Bi}{3+Bi} \qquad (6)$$

At the surface ($T=T_o$) the j-value is lower than 1, at the surface it equals 1, and at approaching the core it is more than 1.

Checking for special cases the j-value becomes as follows:

At the core, y=d:

$$j_k = \frac{3+3[1-½]Bi}{3+Bi} = \frac{1+Bi/2}{1+Bi/3} \quad (1<j_k<1,5)$$

At the surface, y=0:

$$j_o = \frac{3+3[0-0]Bi}{3+Bi} = \frac{1}{1+Bi/3} \quad (o<j_o<1)$$

The average j-value ($y=(1-\sqrt{⅓})d$) becomes:

$$j_g = \frac{3+3[1-\sqrt{⅓} - ½(1-\sqrt{⅓})^2]Bi}{3+Bi} = 1$$

All j-values are in coherence with the special cases, which Levy already calculated.
In full, the temperature distribution can be given as:

$$T(t, \frac{y}{d}) = T_1 + \frac{3+3[\frac{y}{d} - ½(\frac{y}{d})^2]Bi}{3+Bi}(T_b - T_1)\ e^{-t/\tau} \qquad (7)$$

4. <u>CONCLUSIONS</u>

Formulae (6) is been graphically expressed in fig. 5.

For some values of Bi the generalized j-values can be read from a selected value of y. Form (1a) then gives the desired temperature, taking j as the generalized j-value.
It is concluded, that the temperature distribution can indeed be taken as a quare function of depth. Further, the formulation of this generalized j-value is concluded to meet the requirements stated in chapter 1, to arive at a simple way to describe temperature distribution in an infinite slab.

Literature

1. Veerkamp, C.H., Hofmans, G.J.P. Experiments and theoretical calculations relating to processes for cooling of broiler carcasses.
Beekbergen: IPS 't Spelderholt, 1973.
2. Veerkamp, C.H., Hofmans, G.J.P. Het warmtetransport bij luchtkoelen van geslachte kuikens. Koeltechniek 70 (1977) 93-95.
3. Moerman, P.C. Afkoelsnelheid van slachtwarmvlees.
Voedingsmiddelen technologie 5 (1973).
4. Levy, J. Zur theorie der Fleischkühlung. Kältetechikklimatisierung 24 (1972) 85-98.
5. VDI Wärme Altas, Berechnungblätter für den Wärmeübergang, VDI Verlag Düsseldorf 4e Auflage Blatt Ec1.

As a standard example, this publication uses following conditions: a body cools from 40°C in an environment of 0°C at a heat transfer rate of 100 W/m²K.
All variables are as mentioned below:

T_b=40°C c=3360 J/kgK Bi=5
T_l= 0°C ρ=1050 kg/m² τ=45.8 min
d=0,025 m α= 100 W/m²K j_k=1.313
λ=0,5 W/mK j_o=0,375

Fig. 1 Standard cooling process

"Temperature in an infinite slab".

Fig. 2 As a function of depth at the standard process the exact temperature distribution is compared with the square profile.

"Square and exact solution function".

Fig. 3 As a function of depth the temperature difference between exact and square distribution is given at standard conditions (fig. 1). No difference exceeds 0.8°C.

"Temperature distribution in an infinite slab".

Fig. 4 At a certain cooling time the temperature distribution is given.

"Generalized j-value".

Fig. 5From this figure the generalized j-value can be taken as a function of depth at typical Bi-values.