About a problem of modelling of currents of food viscous-plastic media

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A system of equations simulating currents of food viscous-plastic media in channels and cavities of technologital machines and also on elements of their working bodies is obtained. This system of equations differs from the known ones in that it contains ambiguity only in one rheological equation (the basic rheological law of medium deformation). The ambiguity in rheological equation is eliminated due to physical considerations or a mechanical statement of task. The application of the obtained equations for a Bingham medium as an a example is shown.

In the known equations of currents of viscous-plastic media, for example in G. Genki equations, there is an ambiguous expression of intensity of speeds of deformation H [1]. It results in a problem of ambiguity of solution of these equations and complicates modelling by the computer of similar tasks.

For modelling the currents of isotropic, incompressible viscous-plastic medium at isothermal current the following equations are obtained:

$$\frac{dV_x}{dt} = \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) \qquad (1)$$

$$\frac{dV_y}{dt} = \frac{1}{\rho}\left(\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right)$$

$$\frac{dV_z}{dt} = \frac{1}{\rho}\left(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right)$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \qquad (2)$$

$$T = \Phi(H)\,H \qquad (3)$$

$$\frac{2\tau_{xy}}{\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}} = \frac{2\tau_{xz}}{\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x}} = \frac{2\tau_{yz}}{\frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y}} =$$

$$= \frac{\tau_{xx} - \tau_{yy}}{\frac{\partial V_x}{\partial x} - \frac{\partial V_y}{\partial y}} = \frac{\tau_{xx} - \tau_{zz}}{\frac{\partial V_x}{\partial x} - \frac{\partial V_z}{\partial z}} \pm \geq \qquad (4)$$

here:

$$T = \sqrt{\frac{1}{6}\left[(\tau_{xx}-\tau_{yy})^2 + (\tau_{yy}-\tau_{zz})^2 + (\tau_{zz}-\tau_{xx})^2\right] + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2}$$

$$H = \sqrt{\frac{1}{6}\left[(\varepsilon_{xx}-\varepsilon_{yy})^2 + (\varepsilon_{yy}-\varepsilon_{zz})^2 + (\varepsilon_{zz}-\varepsilon_{xx})^2\right] + \varepsilon_{xy}^2 + \varepsilon_{xz}^2 + \varepsilon_{yz}^2},$$

$$\varepsilon_{xx} = \frac{\partial V_x}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial V_y}{\partial y}, \quad \varepsilon_{zz} = \frac{\partial V_z}{\partial z}, \varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2}\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right)$$

$$\varepsilon_{xz} = \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x}\right), \quad \varepsilon_{yz} = \varepsilon_{zy} = \frac{1}{2}\left(\frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y}\right)$$

$\Phi(H)$ --- function dependent on intensity of speeds of deformation

H; $\tau_{xx}, \tau_{yy}, \tau_{zz}, \tau_{xy}, \tau_{xz}, \tau_{yz}$ --- components of a stress tensor T;

$\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xy}, \varepsilon_{xz}, \varepsilon_{yz}$ ---- components of tensor of a speed of deformation currents;

The equations of Bingham media currents are obtained by substitution in a general system of equations instead of the equation (3) the corresponding general rheological relation:

$$T = \tau_0 + 2\mu H \quad (5)$$

Here τ_0 --- limit shear stress; μ --- coefficient of plastic viscosity.

With the help of the obtained new equations the problem about the current of the Bingham medium between parallel planes under the action of constant pressure difference is solved. The problem is solved in a new statement in which there are two areas of current: the area of a shear current at $T > \tau_0$, and the area of a plastic current at $T = \tau_0$ [2]. The conditions at the internal borders of these areas are a continuity of speeds and stresses. The results of the calculation are presented in Fig. 1. The orthographic epure of the normal stress τ_{zz} is not shown in Fig. 1, because it corresponds to rectangular part of the orthographic epure τ_{xx}. The ambiguity at the determination of normal stresses τ_{xx} and τ_{zz} for the area of plastic current is eliminated by the fact that their values are taken negative fore any parameters values that follows from the physical sense of the problem. Thus, it became possible to determine the true character of change of normal stresses τ_{xx} along the cross-section of the current, that other methods did not make it possible to do.

Conclusions

The general equations for research of currents of viscous-plastic media that differ from the earlier ones by the fact that the ambiguity is contained only in one equation – the basic rheological law of deformation of the medium (3) of (5). As the special cases from these equations follow the equations of currents of viscous (Newtonian fluids) and ideally plastic media. These special cases of currents are obtained by equalizing to zero of corresponding rheological constants in the basic rheological law (5) that corresponds to the third axiom of rheology of M. Reiner.

The offered new statement of problems about currents of Bingham media and new obtained equations have allowed to obtain qualitatively new results and at a maximum extent to approach mathematical modelling of such currents to real conditions.

References

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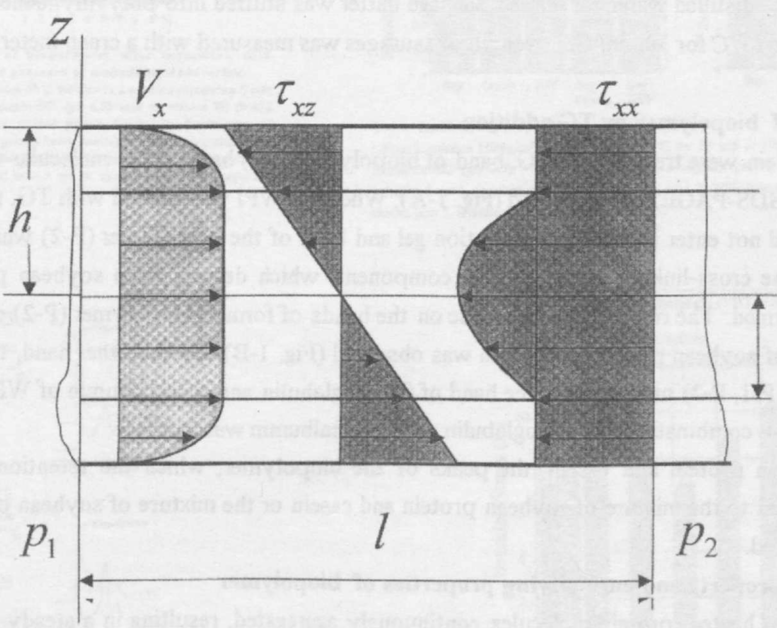

Fig. 1